ÉRETTSÉGI VIZSGA • 2016. május 3.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
    - correct calculation: *checkmark*
    - principal error: *double underline*
    - calculation error or other, not principal, error: *single underline*
    - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
    - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
    - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the markscheme.
2. Subtotals may be **further divided, unless stated otherwise in the markscheme**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
11. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
12. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the markscheme.
13. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

Attention! The **Instructions to examiners** section at the beginning of this marking scheme has changed substantially! Please, read it carefully before starting correction.

I.

1. a) **Solution 1**		
Square both sides of the equation: $\frac{4x^2+44x+121}{9}=x^2+6x+9$.	2 point	
$x^2+2x-8=0$	2 point	
$x = 2$ or $x = –4$.	1 point	
Substitution proves both solutions are correct.	1 point	
Total:	**6 points**	

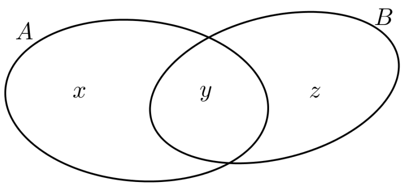
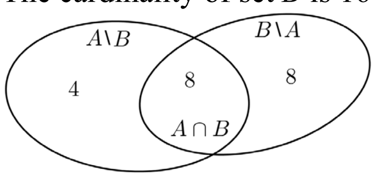
1. a) **Solution 2**		
$\frac{2x+11}{3}=\lvert x+3\rvert$	1 point	
If $x \geq -3$ is true, the equation is $\frac{2x+11}{3}=x+3$,	1 point	
so $x = 2$ (which really is not less than –3).	1 point	
If $x < –3$ is true, the equation is $\frac{2x+11}{3}=-x-3$,	1 point	
so $x = –4$ (which really is less than –3).	1 point	
Check by substitution or refer to the equivalence.	1 point	
Total:	**6 points**	

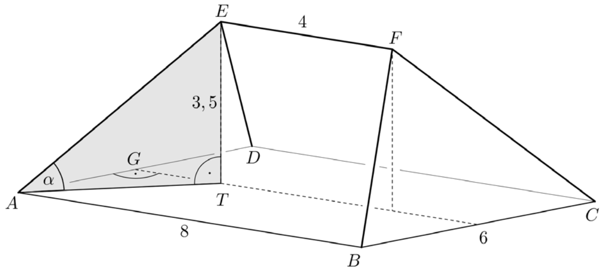
1. b)		
$x > 3$	1 point	*This point is also due if the candidate checks the solution by substitution.*
(Apply the definition and identities of logarithm:) $\log_2\frac{(x+1)(x-3)}{x+9}=$	1 point	
$=\log_2 2$	1 point	*This point is also due if the idea is only made clear in the next step.*
(Due to the monotonicity of the logarithm function:) $\frac{(x+1)(x-3)}{x+9}=2$	1 point	
Rearranged: $x^2-4x-21=0$.	1 point	
The solutions of the quadratic equation are 7 and –3.	1 point	
Check: –3 is incorrect, 7 is a correct solution (use substitution or refer to the domain and the equivalence of steps).	1 point	
Total:	**7 points**	

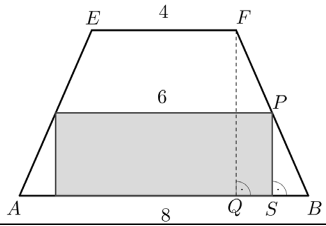
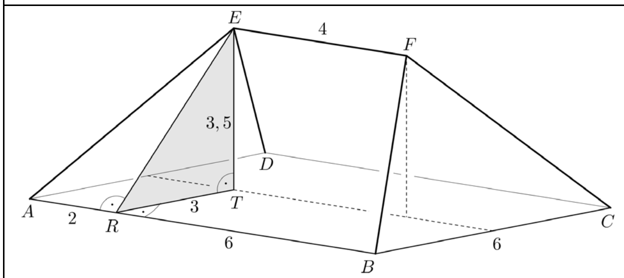
2. a) Solution 1		
5 students got “excellent” in physics,	1 point	
7 students got “excellent” in math,	1 point	
5 + 7 = 12, but only 10 students got “excellent” in at least one of the subjects, so	1 point	
2 students got “excellent” in both subjects.	1 point	
Total:	**4 points**	

2. a) Solution 2		
(Let the number of students receiving “excellent” in both subjects be x. In this case:) $5 - x$ is the number of students receiving an “excellent” grade only in physics, and	1 point	
$7 - x$ is the number of students receiving an “excellent” grade only in mathematics.	1 point	
$5 - x + x + 7 - x = 10$	1 point	
$x = 2$, so 2 students got “excellent” in both subjects.	1 point	
Total:	**4 points**	

Note: Award full points for arriving at the correct result by the use of a correct set diagram.

2. b)		
Make a Venn-diagram.  (Let x, y, and z be the cardinality of sets $A \setminus B$, $A \cap B$, and $B \setminus A$ respectively.)	1 point	
The first term of the arithmetic sequence is x, the second is y, the third is $x + y$, and so	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
the common difference of the sequence is $(x + y) - y = x$.	1 point	
The cardinality of set A is therefore $3x$, that of set B (the fourth term of the sequence) is $4x$.	1 point	
As the sum of these two is 28, so $3x + 4x = 28$.	1 point	
Both the first term and the common difference of the arithmetic sequence is 4.	1 point	
Check: The cardinality of set B is 16 (and so $z = 8$).  $\|A \setminus B\| = 4$, $\| A \cap B \| = 8$, $\| A \| = 12$, $\| B \| = 16$. These four numbers are indeed consecutive terms of an arithmetic sequence.	1 point	
Total:	**7 points**	

3. a)		
 (Refer to the diagram above:) in the right triangle *ATG* *GT* = 2 m, and *AG* = 3 m.	1 point	
(Apply the Pythagorean Theorem:) $AT = \sqrt{13}$ (≈ 3.61) (m).	1 point	$GE = \sqrt{3.5^2 + 2^2} =$ $= \sqrt{16.25}$ (≈ 4.03) (m)
(Apply the Pythagorean Theorem in the right triangle *ATE*:) $AE = \sqrt{AT^2 + ET^2} =$	1 point	$AE = \sqrt{GE^2 + GA^2}$
$= \sqrt{25.25} \approx 5$ (m) is the length of the support beam.	1 point	
The angle between the support beam and the horizontal is angle *EAT* (α in the diagram).	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$\sin\alpha = \dfrac{ET}{AE}$ (≈ 0.6965),	1 point	
$\alpha \approx 44°$.	1 point	
Total:	**7 point**	

3. b)		
The length of one side of the rectangular solar panel may not exceed the length of the midsegment of the trapezium,	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
that is 6 metres.	1 point	
The length of the other side is a constant, equal to half the height of the trapezium (*PS* in the diagram, also the midsegment of triangle *FQB*).	1 point	
 (Refer to the diagram:) the height of the trapezium is calculated from the right triangle *ETR*. $ER = \sqrt{ET^2 + TR^2} = \sqrt{3.5^2 + 3^2} = \sqrt{21.25}$ (≈ 4.61) (m).	1 point	*(Refer to the diagram above:)* $PS = \sqrt{PB^2 - SB^2} =$ $= \sqrt{6.3125 - 1} =$ $= \sqrt{5.3125}$ (≈ 2.30) (m)

The area of the largest solar panel: $\frac{6\cdot\sqrt{21.25}}{2}$ (≈ 13,83) (m²).	1 point	*The area of the largest solar panel:* $6\cdot\sqrt{5.3125}$ (≈ 13.83) (m²)
The area of the largest solar panel that can be attached to the roof is 13.8 m².	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**6 points**	

4. a)		
The total income in case of 1000 tickets sold at 1500 Ft each is 1 500 000 Ft.	1 point	
Increasing the ticket price by 5 Ft n times ($n\in\mathbf{N}^+$) reduces the number of tickets sold to $1000 - 10n$.	1 point	
The income in this case is $(1500 + 5n)(1000 - 10n) =$	1 point	
$= 1\,500\,000 - 10\,000n - 50n^2$ forints,	1 point	
which is less than 1 500 000 Ft,	1 point*	
as positive terms have been subtracted.	1 point*	
Total:	**6 points**	

*The 2 points marked by * may also be given for the following reasoning:*

Let b denote the income as a function of the number of tickets sold. The domain of this function is the set of positive real numbers. In this case $b(n) = -50n^2 - 10\,000n + 1\,500\,000$ $b'(n) = -100n - 10\,000$.	1 point	
The derivative is negative on the entire domain, so function b is strictly monotone decreasing (and so it remains, too, when restricted to the domain of positive integers only).	1 point	

4. b)		
If the ticket price is decreased by 5 Ft m times, the number of tickets sold will be $1000 + 10m$ ($m\in\mathbf{Z}$).	1 point	
The total income in this case is $(1500 - 5m)(1000 + 10m) =$	1 point	
$= 1\,500\,000 + 10\,000m - 50m^2$ (Ft).	1 point	
Completing the square: $-50(m - 100)^2 + 2\,000\,000$, so	2 points*	
the maximum is at $m = 100$.	1 point*	
The tickets price in this case is $(1500 - 100\cdot 5 =)$ 1000 Ft.	1 point	
The maximal income is $(1000\cdot 2000 =)$ 2 000 000 Ft.	1 point	
Total:	**8 points**	

*The 3 points marked by * may also be given for either of the following reasoning:*

Let f denote the income as a function of the number of tickets sold. The domain of this function is the set of positive real numbers. In this case $f(m) = -50m^2 + 10\,000m + 1\,500\,000$ $f'(m) = -100m + 10\,000$.	1 point	
If $f'(m) = 0$, then $m = 100$.	1 point	
As $f''(m) = -100 < 0$, at 100 there is in fact a maximum of function f (and also of the function restricted to the domain of integers).	1 point	*This point is also due if the candidate correctly refers to the first derivative changing its sign.*

Let f denote the income as a function of the number of tickets sold: $f(m) = -50m^2 + 10\,000m + 1\,500\,000$. The domain of this function is the set of positive real numbers. The zeros of function f are 300 and –100.	1 point	
Function f has a maximum which it attains at the arithmetic mean of the two zeros.	1 point	
The maximum of function f is therefore attained at 100. It is also where the total income is maximal.	1 point	

II.

5. a) Solution 1		
The number of defective shirts made on the first machine is 80, while on the second it is 170.	1 point	
(As there are a total 250 defective shirts) the number of possible choices is 250.	1 point	*These 2 points are also due if the correct reasoning is reflected by the solution.*
(170 defective shirts were made on the second machine and so) the number of favourable choices is 170.	1 point	
The probability is $\frac{170}{250} =$	1 point	
$= 0.68$.	1 point	*The correct answer is acceptable in percentage form, too.*
Total:	**5 points**	

5. a) Solution 2		
The number of defective shirts made on the first machine is 80, while on the second it is 170.	1 point	
Let A be the event that the selected shirt was made on the second machine. Let B be the event that the selected shirt is defective. The probability then is: $P(A\|B)=\frac{P(AB)}{P(B)}$.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$P(AB)=\frac{170}{9000}=\frac{17}{900}$	1 point	
$P(B)=\frac{250}{9000}=\frac{25}{900}$	1 point	
$P(A\|B)=\frac{\frac{17}{900}}{\frac{25}{900}}=\frac{17}{25}=0.68$	1 point	*The correct answer is acceptable in percentage form, too.*
Total:	**5 points**	

5. b)		
Let x be the price of the shirt before it went on sale, and let $q=1-\frac{p}{100}$.	1 point	
(Had the discounts followed each other in reverse order, the final price of the shirt would have been $xq-500$ forints, and therefore) $(xq-500)+50=(x-500)\cdot q$.	2 points	
(After decreasing the price by p% twice, the final price will be xq^2 forints, and therefore) $(x-500)\cdot q+90=xq^2$.	2 points	
From the first equation $q = 0.9$,	1 point	
that is $p = 10$.	1 point	
(Substituting q into the second equation) $(x-500)\cdot 0.9+90=x\cdot 0.81$.	1 point	
$x = 4000$	1 point	
The original price of the shirt was 4000 Ft, $p = 10$.	1 point	
Check: 4000 – 500 = 3500, 10% discounted it is 3150; 4000 discounted by 10% is 3600, 3600 – 500 = 3100; 4000 discounted by 10% is 3600, discounted by 10% again is 3240. 3150 = 3100 + 50 and 3150 = 3240 – 90, so the solutions are correct.	1 point	*Award this point only if the candidate checks the solution against the text of the original problem.*
Total:	**11 points**	

6. a)		
From the equation $\cos x+2=\sin x+2$: $x=\frac{\pi}{4}+k\pi$ ($k\in\mathbf{Z}$).	1 point	*Do not award this point if the candidate only reads the endpoints of the interval off the graph, without any calculation and checking by substitution.*
The first coordinate of the points of intersection of the two graphs will be $-\frac{3\pi}{4}$, and $\frac{\pi}{4}$.	1 point	
$T=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\left((\cos x+2)-(\sin x+2)\right)dx=$	2 points	$T=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}(\cos x+2)dx-$ $-\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}(\sin x+2)dx=$
$=[\sin x+\cos x]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}=$	2 points	$=[\sin x+2x]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}-$ $-[-\cos x+2x]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}$
$=\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)-\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)=$	1 point	$\left(\frac{\sqrt{2}}{2}+\frac{\pi}{2}\right)-\left(-\frac{\sqrt{2}}{2}-\frac{3\pi}{2}\right)-$ $-\left[\left(-\frac{\sqrt{2}}{2}+\frac{\pi}{2}\right)-\left(\frac{\sqrt{2}}{2}-\frac{3\pi}{2}\right)\right]=$ $=\left(\sqrt{2}+2\pi\right)-\left(-\sqrt{2}+2\pi\right)=$
$=2\sqrt{2}$ (≈ 2.83)	1 point	
Total:	**8 points**	

6. b) Solution 1		
$a_1=-\frac{6}{5}$, $a_2=-\frac{17}{2}$, $a_3=28$.	1 point	
As $a_2<a_1<a_3$, the sequence is not monotonic.	1 point	
If $n\geq 4$, then $a_n>0$ (since both the numerator and the denominator of the fraction are positive), so the sequence has a lower bound (a possible lower bound is $-\frac{17}{2}$).	1 point	

(Rearranging the rule defining the sequence:) $a_n = \dfrac{11-\frac{5}{n}}{3-\frac{8}{n}}$.	1 point	$a_n = \dfrac{11n-5}{3n-8} = \dfrac{\frac{11}{3}\cdot(3n-8)+\frac{73}{3}}{3n-8} =$
If $n \geq 4$, then the numerator is less than 11 and the denominator is at least 1,	1 point	$= \dfrac{11}{3} + \dfrac{73}{9n-24}$
and so $a_n < \dfrac{11}{1} = 11$.	1 point	*If $n \geq 4$, then it is less than 10* ($a_n \leq \dfrac{11}{3} + \dfrac{73}{12} = \dfrac{39}{4} < 10$).
(Both $a_2 < a_3 = 28$ and $a_1 < a_3 = 28$ are true, so) 28 is an upper bound of the sequence.	1 point	
As the sequence has both upper and lower bounds it is indeed bounded.	1 point	
Total:	**8 points**	

6. b) Solution 2		
$a_1 = -\dfrac{6}{5}$, $a_2 = -\dfrac{17}{2}$, $a_3 = 28$.	1 point	
As $a_2 < a_1 < a_3$, the sequence is not monotonic.	1 point	
Rearrange the rule defining the sequence $\{a_n\}$: $a_n = \dfrac{11-\frac{5}{n}}{3-\frac{8}{n}}$.	1 point	$a_n = \dfrac{11n-5}{3n-8} = \dfrac{\frac{11}{3}\cdot(3n-8)+\frac{73}{3}}{3n-8} =$
The sequence $\{b_n\}$ is defined by the formula in the numerator of the fraction. This sequence is convergent (its limit is 11).	1 point	$= \dfrac{11}{3} + \dfrac{73}{9n-24}$
The sequence $\{c_n\}$ is defined by the formula in the denominator of the fraction. This sequence is convergent (its limit is 3).	1 point	*If $d_n = \dfrac{73}{9n-24}$, then the sequence $\{d_n\}$ is convergent, too (its limit is 0),*
Since both sequences $\{b_n\}$ and $\{c_n\}$ are convergent, their ratio, the sequence $\{a_n\}$ is convergent, too. (Its limit is $\dfrac{11}{3}$).	1 point	*so the sequence $\{a_n\}$ is convergent, too (its limit is $\dfrac{11}{3}$).*
All convergent sequences are necessarily bounded,	1 point	
so the convergent sequence $\{a_n\}$ is bounded, too.	1 point	
Total:	**8 points**	

7. a) Solution 1		
There are 9^3 (= 729) three-digit numbers that do not contain 0 as a digit.	1 point	
There are 8^3 (= 512) among these that do not contain 1 either.	1 point	
There are as many as $9^3 - 8^3$ (= 217) three-digit numbers satisfying all conditions.	1 point	
The number of two-digit numbers satisfying all conditions is similarly $9^2 - 8^2$ (= 17),	1 point	
and there is a single one-digit number, 1 itself.	1 point	
The total number of appropriate positive integers is the sum of the above: $9^3 - 8^3 + 9^2 - 8^2 + 1$ (= 235).	1 point	
Total:	**6 points**	

7. a) Solution 2		
Among those three-digit numbers that do not contain zero, there are $3 \cdot 8 \cdot 8$ (= 192) that contain exactly one 1.	1 point	
$3 \cdot 8$ (= 24) of the above contain two 1-s, while	1 point	
the only number containing three 1-s is 111.	1 point	
Among those two-digit numbers that do not contain zero, there are $2 \cdot 8$ (= 16) that contain exactly one 1, while the only number containing two 1-s is 11. This adds up to 17 different two-digit numbers.	1 point	
The only appropriate one-digit number is 1.	1 point	
The total number of appropriate positive integers is the sum of the above, 235.	1 point	
Total:	**6 points**	

7. b)		
If the (only) number *m* is replaced by *m* + 10, (the number of data will not change and) the sum of the data will increase by 10.	1 point	$22n + 10 = 24n$
The mean increases by 2, which means the number of data is $\left(\frac{10}{2} = \right) 5$.	1 point	$n = 5$
Total:	**2 points**	

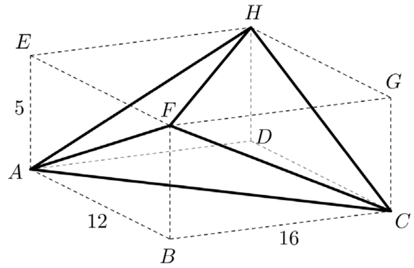
7. c)		
The sum of the 5 numbers in the original set of data is $(5 \cdot 22 =)$ 110.	1 point	
The mode is 32, which means the frequency of 32 is at least 2.	1 point	
Arranging the 5 data in non-decreasing order, the number just before median m is $m - 4$ (as decreasing m by 5 would replace m as median).	2 points*	*$m - 4 = 10$ is not possible (the sum of the 5 data wouldn't be 110).*
The lowest of the data is 10, so $10 + (m - 4) + m + 32 + 32 = 110$.	1 point*	
Which gives $m = 20$.	1 point*	
The data must be: 10, 16, 20, 32, 32.	1 point	
Check: the mean of these data is 22; replacing 20 by 30 changes the mean to 24; replacing 20 by 15 changes the median to 16.	1 point	
Total:	**8 points**	

*The 4 points marked by * may also be given for the following reasoning:*

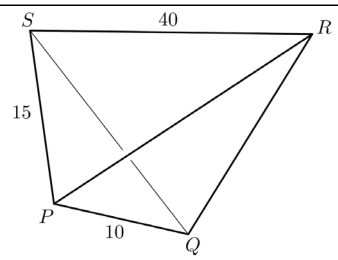
Among the data there is 10 (which is also the lowest) and also, at least twice, there is 32.	1 point	
The sum of the two data still unknown: $110 - (10 + 32 + 32) = 36$.	1 point	
These two data are both positive integers, so the possible pairs are: $10 + 26 = 11 + 25 = ... = 17 + 19$.	1 point	
The only suitable pair is 16 and 20.	1 point	

8. a)		
The volume of tetrahedron *ACFH* may be obtained by subtracting the volumes of the four congruent pyramids at vertices *B, D G,* and *E* from the volume of the cuboid.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The volume of one such pyramid is $\frac{5 \cdot 12 \cdot 16}{6} = 160$ (cm^3).	1 point	
The volume of tetrahedron *ACFH* is $5 \cdot 12 \cdot 16 - 4 \cdot \frac{5 \cdot 12 \cdot 16}{6} =$	1 point	
$= 320$ cm^3.	1 point	
Total:	**4 points**	

Note: Do not award points if the candidate only calculates the area of one triangular face of tetrahedron ACFH.

8. b)		
 Diagram (correctly representing the tetrahedron within the cuboid).	1 point	*This point is also due if the candidate gives the correct answer without the use of a diagram.*
The diagonals of congruent rectangles are equal: $AC = FH$, $AF = CH$ and $AH = CF$.	1 point	
The faces of the tetrahedron are therefore triangles, whereby corresponding sides are equal in pairs, and so these triangles are congruent.	1 point	
Total:	**3 points**	

8. c)		
To prove the lateral triangles are acute, it is enough to show that the greatest angle of one such triangle is acute.	1 point	*This point is also due if the candidate correctly calculates all three angles (83.4°; 56.4°; 40.2°)*
E.g. in triangle AFH (applying the Pythagorean Theorem) $AF = 13 < AH = \sqrt{281} < FH = 20$,	1 point	
so the cosine of angle φ opposite side FH of the triangle (applying the Law of Cosines) is $\cos\varphi = \frac{169+281-400}{2\cdot 13\cdot\sqrt{281}}$ (≈ 0.1147).	2 points	
As this is a positive number, angle φ is acute indeed.	1 point	
Total:	**5 points**	

8. d)		
In triangle PRS $PS = 15$ cm and $SR = 40$ cm, so (according to the triangle-inequality, and given that the length can only be 20, 25, or 30 cm) the only possible length is $PR = 30$ cm.	1 point	
In triangle PQS $PS + PQ = 25$, so QS can only be 20 cm.	1 point	
Therefore, QR must be 25 cm, and (as $20 + 25 > 40$) triangle RSQ is also an existing one.	1 point	
Only one tetrahedron meets the given criteria.	1 point	
Total:	**4 points**	

9. a) Solution 1		
Field 4 will be reached in either one, two, three, or four steps.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The only way to reach field 4 in one step is by rolling 4, the probability of which is $\frac{1}{6}$.	1 point	
In two steps: roll either 3-1, 2-2 or 1-3.	1 point	
The combined probability of these is $\left(3\cdot\frac{1}{6}\cdot\frac{1}{6}=\right)\frac{3}{36}$.	1 point	
In three steps: roll 1-1-2, 1-2-1 or 2-1-1.	1 point	
The combined probability of these is $\left(3\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\right)\frac{3}{216}$.	1 point	
In four steps: you must roll four 1-s, the probability of which is $\left(\frac{1}{6}\right)^4=\frac{1}{1296}$.	1 point	
The probability of arriving at field four at least once during the game is the sum of the above probabilities,	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$\frac{343}{1296}\approx 0.265$.	1 point	*The correct answer is acceptable in percentage form, too.*
Total:	**9 points**	

9. a) Solution 2		
On the first roll, we may have got a 4, 3, 2, or 1.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
If we got 4 on the first roll that immediately took us to field 4. The probability of this is $\frac{1}{6}$.	1 point	
If we got 3 on the first roll, the only way to get to field 4 is by rolling a 1 for the second time. The probability of this is $\left(\frac{1}{6}\right)^2=\frac{1}{36}$.	1 point	
If we got a 2 on the first roll, we must continue with a 2 or two 1-s.	1 point	
The probability of this is $\left(\frac{1}{6}\right)^2+\left(\frac{1}{6}\right)^3=\frac{1}{36}+\frac{1}{216}$.	1 point	

If we got a 1 on the first roll, we may proceed by any of the following combinations: 3; 2-1; 1-2; 1-1-1.	1 point	
The probability of this is $\left(\frac{1}{6}\right)^2+2\cdot\left(\frac{1}{6}\right)^3+\left(\frac{1}{6}\right)^4=\frac{1}{36}+\frac{2}{216}+\frac{1}{1296}$.	1 point	
The probability of arriving at field 4 at least once during the game is the sum of the above probabilities,	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$\frac{343}{1296}\approx 0.265$.	1 point	*The correct answer is acceptable in percentage form, too.*
Total:	**9 points**	

9. a) Solution 3		
Calculate the probability of the complement event. If we never arrived at field 4, we must have skipped it on the first, second, third, or fourth roll.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
To skip it on the first roll, we must have rolled a 5 or a 6. The probability of this is $\frac{2}{6}$.	1 point	
To skip it on the second roll, our first two rolls must have been one of the following: 1-4, 1-5, 1-6, 2-3, 2-4, 2-5, 2-6, 3-2, 3-3, 3-4, 3-5, 3-6. The probability of this is $\frac{12}{36}$.	2 points	*Grouped according to the sum of the rolls:* 1-4, 2-3, 3-2; 1-5, 2-4, 3-3; 1-6, 2-5, 3-4; 2-6, 3-5; 3-6.
To skip it on the third roll, we have 5 options for a third roll after a 1-2 or a 2-1 (as, for the third, we must have rolled at least a 2), and 4 options after a 1-1 (as, for the third, we must have rolled at least a 3). The probability of this is $\frac{14}{216}$.	2 points	2-1-2, 1-2-2, 1-1-3; 2-1-3, 1-2-3, 1-1-4; 2-1-4, 1-2-4, 1-1-5; 2-1-5, 1-2-5, 1-1-6; 2-1-6, 1-2-6.
To skip it on the fourth roll, the first three must have been 1-1-1, while the fourth could have been any of five different numbers (at least 2). The probability of this is $\frac{5}{1296}$.	1 point	
To get the probability of arriving at field 4 at least once during the game, the sum of the above probabilities must be subtracted from 1.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
This probability is therefore $1-\left(\frac{2\cdot 216+12\cdot 36+14\cdot 6+5}{1296}\right)=\frac{343}{1296}\approx 0.265$.	1 point	*The correct answer is acceptable in percentage form, too.*
Total:	**9 points**	

9. b) **Solution 1**		
Examine different possibilities according to the number of times András got a 4 on his first three rolls.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
Got 4 three times: only 1 possibility.	1 point	
Got 4 twice: this is not possible as, in this case, he must also have got a 4 on his third roll.	1 point	
Got 4 once: this could happen on either the first or the third roll.	1 point	
The sum of the other two rolls is also four (1-3, 3-1, or 2-2); this gives three possibilities either way, a total of 6.	1 point	
András never got a 4 on his first three rolls: he must have rolled 1-1-2 in one or the other order, a total 3 possibilities.	1 point	
The ultimate number of possibilities for the three rolls is therefore 10.	1 point	
Total:	**7 points**	

9. b) **Solution 2**		
Before his fourth roll, András could have arrived to field 4 for the first, second, or third time.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
Assuming this is his first, his possible combinations are 1-1-2, 1-2-1, or 2-1-1. This gives three options.	1 point	
Assuming this is his second time, his first could have been on either the first or the second roll.	1 point	
If he got to field 4 for the first time on his first roll then this roll must have been a 4, and the other two are either 3-1, 1-3, or 2-2. This gives another three options.	1 point	
If he got to field 4 for the first time on his second roll then his first two rolls must have been either 3-1, 1-3, or 2-2, while the third roll must have been a 4. This gives three options again.	1 point	
Assuming he got to field 4 for the third time in three rolls, all of those rolls must have been 4-s. This is one more option.	1 point	
The total number of different options is 10.	1 point	
Total:	**7 points**	

9. b) **Solution 3**		
András's first roll could have been a 4, 3, 2, or 1.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
Assuming his first roll was a 4, the others (arranged in decreasing order) could have been either 4-4-4, 4-3-1, 4-2-2, 4-1-3. This gives 4 different options.	2 points	
Assuming his first roll was a 3, the only possible way to continue would have been 3-1-4. This is one more option.	1 point	
Assuming his first roll was a 2, the sequence could have been 2-2-4, or 2-1-1. This gives two more options.	1 point	
Assuming his first roll was a 1, the sequence could have been 1-3-4, 1-2-1, or 1-1-2. This gives another three options.	1 point	
There are a total 10 different options for the first three rolls.	1 point	
Total:	**7 points**	

Notes:

I) Award 6 points if the candidate correctly and completely lists all 10 possible options in some logical order (i.e. a systematic approach is recognisable), but without any reasoning.

II) Award a maximum of 4 points if the candidate correctly and completely lists all 10 possible options without any logical order (no systematic approach is recognisable) and without reasoning. (In this case it is not made clear why there may not be any other options.)

Use the table below to determine the number of points to be awarded if the candidate lists possible options without reasoning but certain errors are made (an option is missed or listed more than once).

Number of errors	*score in case I)*	*score in case II)*
1	5	3
2	4	2
3	3	1
4	2	0
5	1	0
6 or more	0	0